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THE γ-MODEL: A SIMPLE HYSTERETIC MODEL FOR REINFORCED CONCRETE WALLS

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1 INTRODUCTION

Non-linear time-history analysis for seismic evaluation or design requires reliable ground acceleration time-histories for the seismic input and reliable models for the structural behaviour. This paper addresses the second issue, more specifically the definition of non-linear force-displacement relationships, called hysteretic model, for the simulation of non-linear behaviour. The sensitivity of the computed results to the characteristics of the input time-history is a well known characteristic of non-linear seismic analysis. In order to account for this variability, the determination of the seismic response must be repeated several times with different ground acceleration time-histories. In this context, the complexity of the selected hysteretic model controls the required computing time investment. It is therefore interesting to be able to use a simple and realistic hysteretic model for an efficient calculation effort.

The γ-model is a new hysteretic model developed for reinforced concrete (RC) structural walls to accurately simulate the seismic behaviour with a limited computational effort. The model is based on the measured non-linear responses of six RC structural walls recorded during dynamic tests. By contrast to static-cyclic tests, dynamic tests provide realistic seismic response such as the one which occurs during a real earthquake. Sample hysteretic loops for two test walls are plotted in Figure 1 (see section 3 for background information on the tests). The empirical observation that the reloading curves tend to cross at the same point ("cross point" spotted through circles in the plots) constitutes the basis of the γ-model.

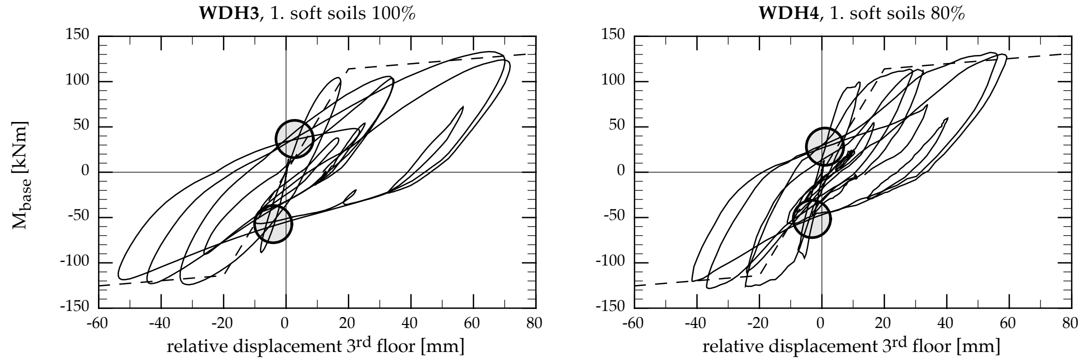


Fig. 1. Hysteretic loops of two test walls. Circles spot the "cross point" of the reloading curves which is at the base of the γ-model.

Comparisons with three established hysteretic models are performed to assess the reliability of the γ-model. The hysteretic models are, in order of increasing complexity, the classic elastoplastic model, the Q-model and the Takeda-model. The comparisons of the different hysteretic models are focused on the accurate reproduction of the measured relative displacements at the top of the test wall and on the correct prediction of the ductility demand.

2 ESTABLISHED HYSTERETIC MODELS

The three established hysteretic models with bi-linear envelope used in this investigation are briefly described in this section. A complete description may be found in the related references. For further insight about force-deformation relationships for RC elements, a review of the most important hysteretic models suitable for reinforced concrete may be found in [1].

2.1 Elastoplastic model

The elastoplastic model (EP-model), sometimes also called bi-linear model, is shown in Figure 2. Even if it is mainly intended for elastoplastic material, such as steel, this model is intensively used for all types of materials because of its simplicity.

The force-displacement relationships of the EP-model are totally specified through three parameters: the stiffness (K), the yield displacement (u_y) and the post yield stiffness expressed as a portion (r K) of the stiffness. The displacement ductility is expressed as the ratio (u_p/u_y) of the peak displacement (u_p) to the yield displacement (u_y). For the simulation of reinforced concrete, the main default of the EP-model is the too stiff reloading curve after yielding and unloading. This characteristic does not take into account the closure of the cracks. It leads to excessive energy dissipation by the inelastic cycles and to unrealistic permanent deformations.

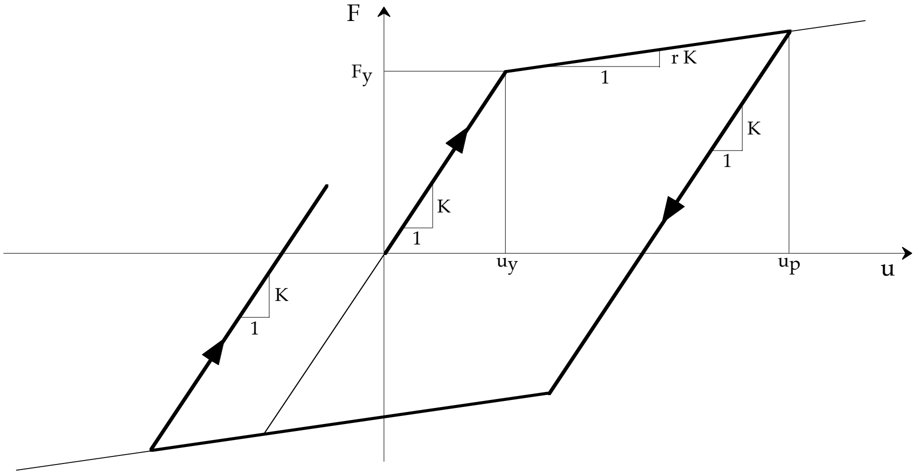


Fig. 2. Force-displacement relationships defining the elastoplastic model.

2.2 Takeda model

Since the Takeda-model includes realistic conditions for the reloading curves, it provides a much better simulation of the features of reinforced concrete in comparison with the EP-model. Moreover, the Takeda-model takes into account the degradation of the stiffness due to increasing damage, which is an important feature of reinforced concrete subjected to seismic action [1]. However, the Takeda-model does not include strength degradation.

The Takeda-model was initially proposed in a basic version by Takeda et al. [2] and was adapted afterwards through many authors. The version used here is the one of Allahabadi and Powell [3]. The force-displacement relationships of the Takeda-model are specified through five parameters: the initial stiffness (K_0), the yield displacement (u_y), the post yield stiffness (r K_0), α relating the stiffness degradation and β specifying the target for the reloading curve. Different rules are used for large and for small hysteretic cycles. The small cycles are, one more time, divided into small cycles with yielding and small cycles with small amplitudes. The force-displacement relationships for large cycles are shown in Figure 3 and the ones for small cycles are shown in Figure 4. All these specific rules for the different hysteretic cycles significantly increase the complexity of the Takeda-model for its implementation. It is our experience that under certain combination of parameters and certain characteristics of the input time-history, the Takeda-model is sometimes numerically unstable.

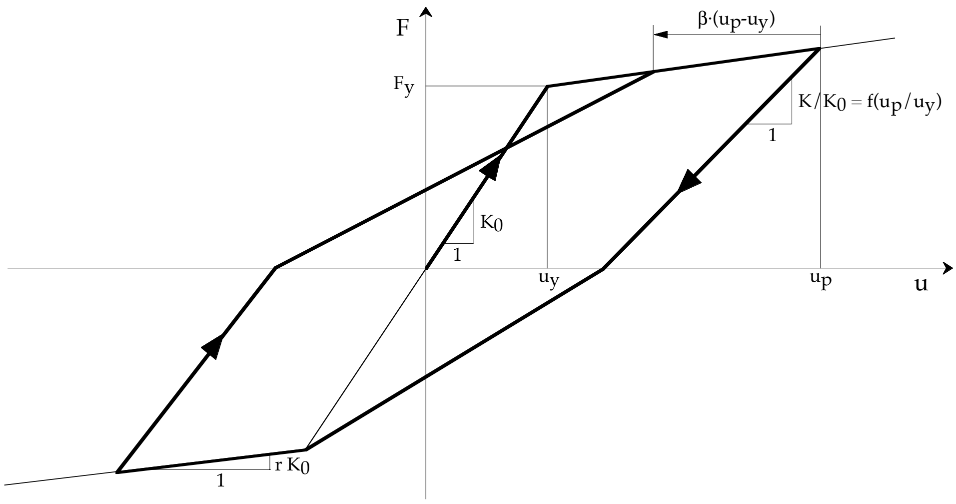


Fig. 3. Force-displacement relationships defining the large inelastic cycles of the Takeda-model.

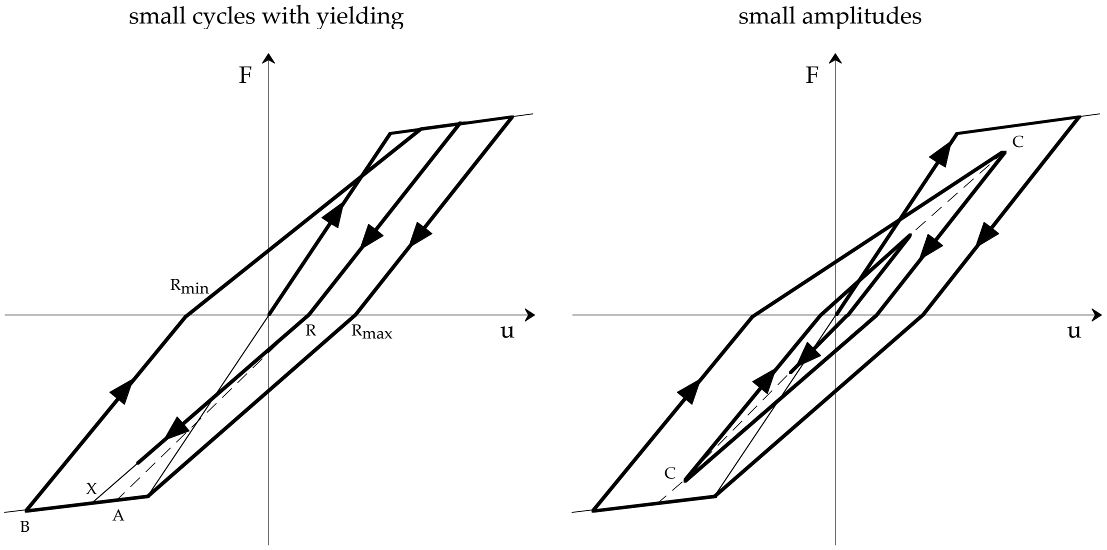


Fig. 4. Force-displacement relationships defining the small hysteretic cycles of the Takeda-model.

2.3 Q-Model

A simplified version of the Takeda-model was proposed by Saiidi and Sozen [4], the Q-model. The force-displacement relationships of the Q-model are shown in Figure 5. In comparison with the Takeda-model, the consideration of the absolute value of peak displacement for both directions constitutes the main simplification. Moreover, there are no distinctions between large and small hysteretic cycles. The reloading curves systematically target the point corresponding to the absolute value of actual peak displacement ($|u|_{max}$).

Similar to the Takeda-model, the Q-model takes into account the stiffness degradation, but does not take into account the strength degradation. The force-displacement relationships of the Q-model are totally specified through four parameters: the initial stiffness (K_0), the yield displacement (u_y), the post yield stiffness ($r\ K_0$) and α relating the stiffness degradation.

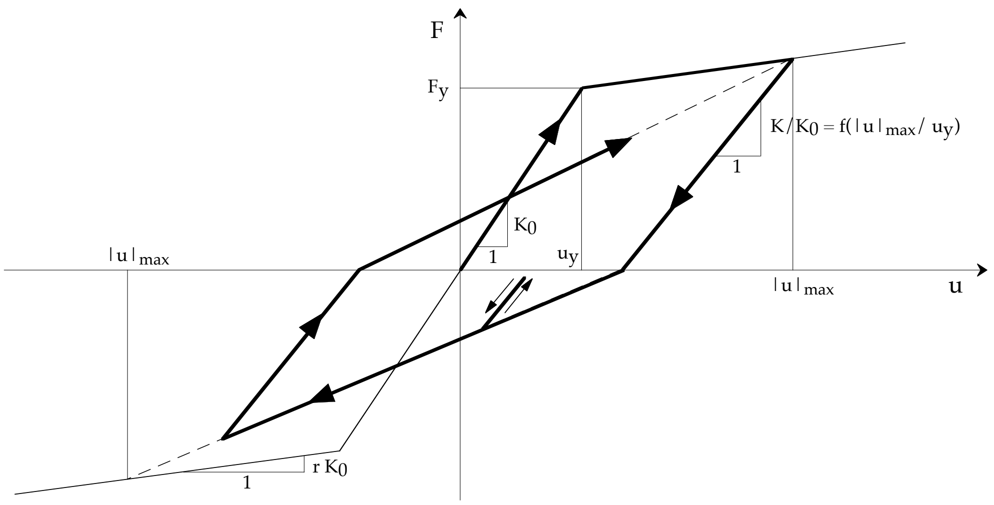


Fig. 5. Force-displacement relationships defining the Q-model.

3 DYNAMIC TESTS

The dynamic tests were performed using the ETH earthquake simulator (shake table) at the Institute of Structural Engineering (IBK) of the Swiss Federal Institute of Technology (ETH) in Zurich, Switzerland [5]. Six walls, labelled WDH1 to WDH6, were tested. As shown in Figure 6, the test walls were designed to be representative at 1:3 scale of a structural wall of a three-storey reinforced concrete reference building. The reference building is a typical structural wall system consisting of flat slabs, slender columns designed for gravity loads only and a few relatively slender structural walls of rectangular cross-section.

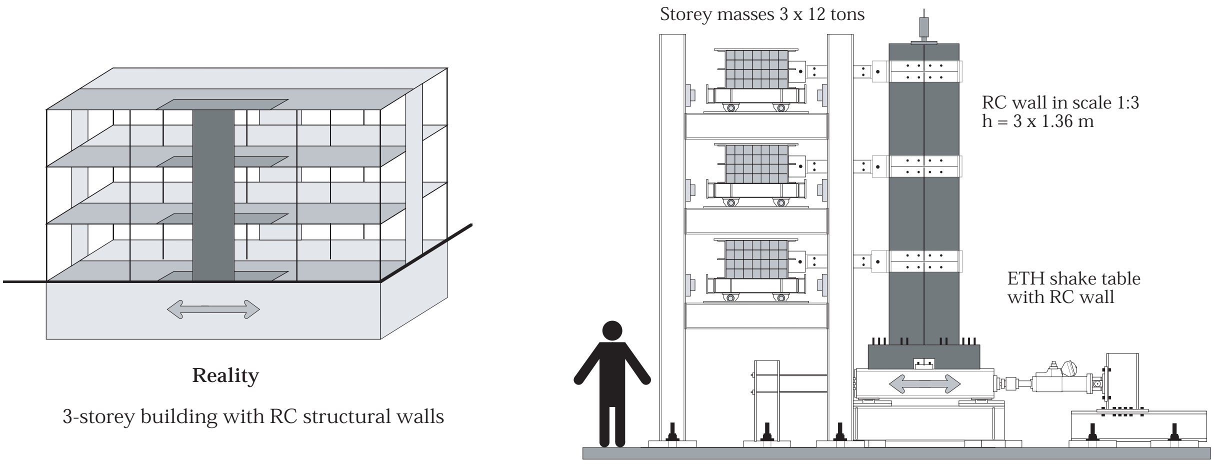


Fig. 6. Reference building with structural wall system and the corresponding reinforced concrete test wall in scale 1:3 with test set-up.

3.1 Test set-up

The test set-up simulates the main features of the 3-storey reference building shown in Figure 6 (left). In order to obtain the desired relation between tributary areas of gravity loads and horizontal inertial forces, a separate structure was developed with three horizontally moveable masses of 12 t attached to the wall by pinned steel struts (Figure 6 right). This test set-up therefore permitted a realistic simulation of the reference building. Only the favourable and relatively small stiffness resulting from the frame effect of the slabs and columns with the structural walls in the real building was neglected. The wall footing was rigidly connected to the shake table, which operates in one horizontal direction and may move up to 125 mm in each direction [6]. The axial force due to gravity loads at the base of the wall (plastic region) was applied by external post-tensioning. The resulting time-histories of dynamic bending moments and shear forces together with the axial force simulate well the stress situation in the wall during a real earthquake.

3.2 Test walls

The test walls were rectangular in cross-section with the following dimensions: horizontal wall length l_w= 0.90 m, wall width b_w=0.10 m and total height including footing h_{tot}=4.65 m. The related aspect ratio of 4.5:1 leads to a wall behaviour dominated by flexure. The concrete dimensions and the reinforcement were chosen considering the available bending moment reaction capacity of the earthquake simulator [6]. The design and detailing were based on the capacity design method [7]. Concerning reinforcement ratio, the total vertical reinforcement ranged between 0.47% and 0.60%. The axial force was equal to 3% of the resistance of the gross section for all walls and the maximum nominal shear stress was between 0.74 MPa and 0.93 MPa.

3.3 Dynamic excitations

A synthetic spectrum-compatible earthquake was used for the tests in order to facilitate the comparison of the results with the codes' assumptions. The synthetic earthquake was generated using a stationary simulation. Figure 7 shows the ground motion used in the tests. It is compatible with the design spectrum Eurocode 8 (ENV 1998-1-1 to 3) [8] for soft soils and a peak ground acceleration of 1.6 m/s^2. The acceleration time-history (left) and the related acceleration response spectra (right) are plotted. The corner frequencies of 1.25 Hz and 5 Hz define the plateau with a constant spectral acceleration of 3.6 m/s^2. The earthquake lasts approximately 14 seconds.

Each wall was subjected to several tests. An 80% earthquake means that the accelerations of the test excitation reached 80% of the accelerations of the reference synthetic earthquake shown in Figure 7.

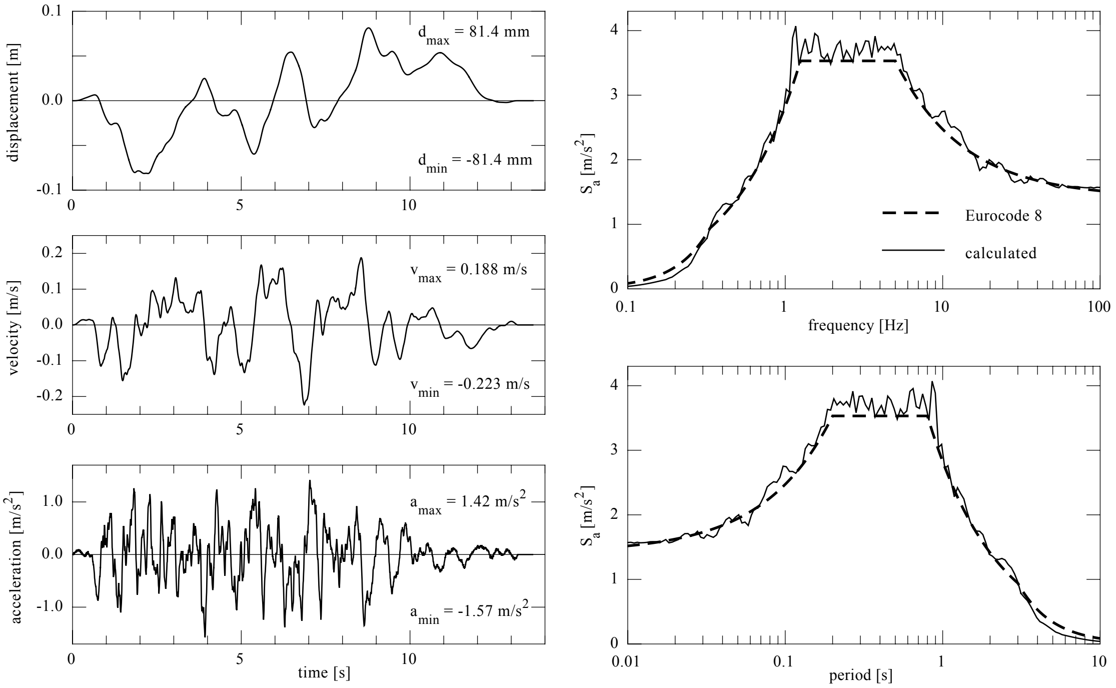


Fig. 7. Synthetic earthquake compatible with Eurocode 8' s design spectrum for soft soils.

3.4 Test results

The objective of the tests was to analyse the dynamic non-linear behaviour of reinforced concrete walls under seismic excitations. The test results were intended to calibrate input parameters of numerical models and to check existing design rules of structural walls. Selected results are presented in this section. Complete results of the dynamic tests are published in [5] and [9], and related interpretations are published in [10]. The test data are available on the web site of the Institute of Structural Engineering of ETHZ [5].

The time-histories of the measured relative displacements at the 3rd floor (left) and the related hysteretic loops (right) for the first test of WDH3 and the first two tests of WDH5 are plotted in Figure 8. The peak values of the relative displacements and the related times are indicated for both directions. Relative displacements up to 73 mm were measured at the 3rd floor, corresponding to an average storey drift of more than 1.8%. Note that, the flexural strength of WDH5 was approximately 30% higher than the one of WDH3. Despite these different strengths, the measured relative peak displacements for the 100% earthquakes were remarkably similar (between 71.3 mm and 73.0 mm). As discussed in a companion paper, this confirms the equal displacement rule [11].

Concerning hysteretic loops, the bending moment at the base of the wall is plotted against the measured relative displacement at the 3rd floor. Even if significant inelastic deformations were reached, the measured hysteretic loops were stable. They presented practically no decrease of the resistance and limited pinching. The displacement ductility ($\mu_{\Delta,m}$) reached during the tests was determined using yield displacement derived from the measurements [10]. It was estimated around $\mu_{\Delta,m}$=3.4 for WDH3 with a 100% earthquake. For WDH5, the displacement ductility was estimated around $\mu_{\Delta,m}$=2.0 for the first test with a 80% earthquake and around $\mu_{\Delta,m}$=3.2 for the second test with a 100% earthquake.

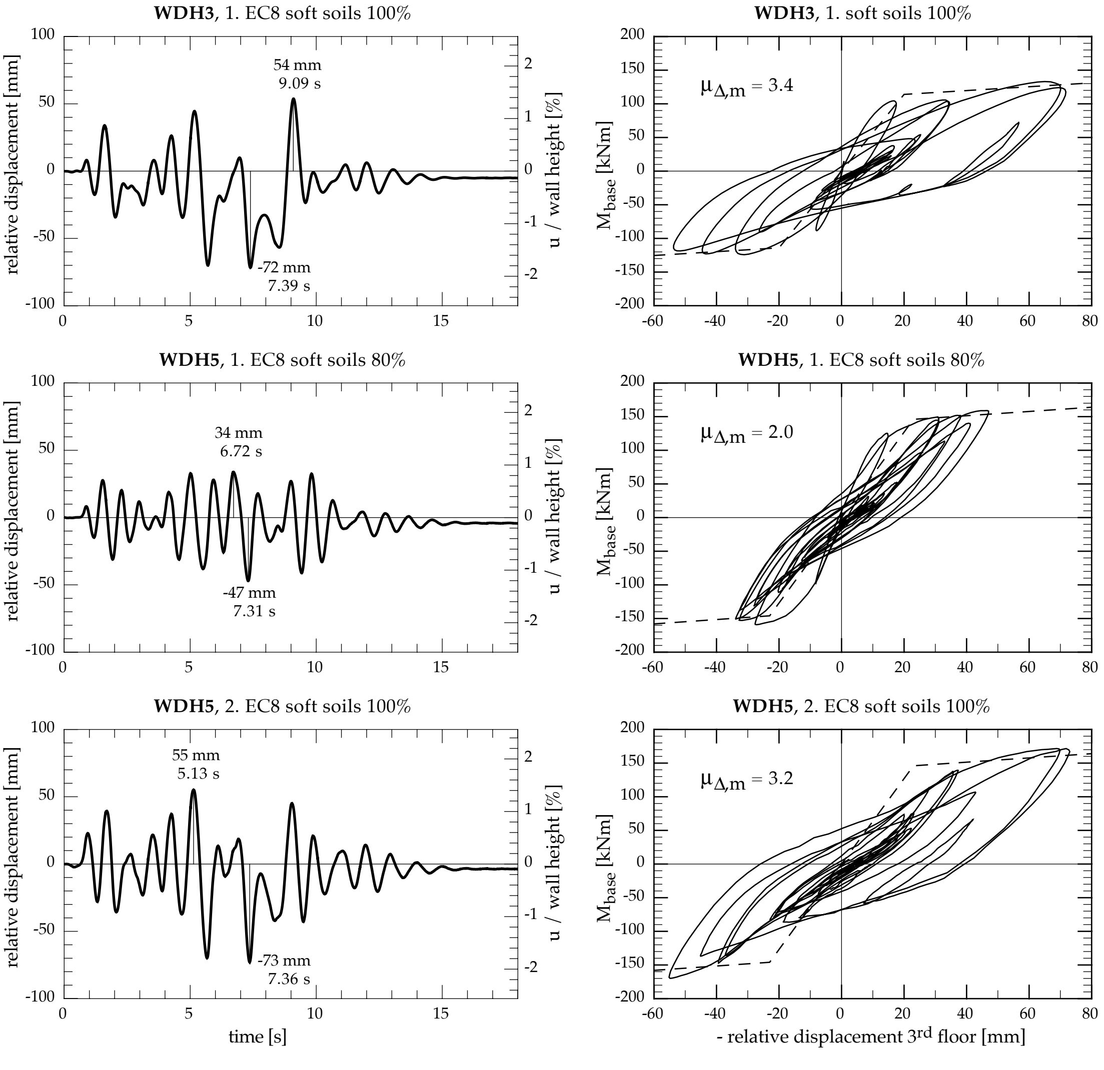


Fig. 8. Time-histories of the measured relative displacements at the 3rd floor (left) and hysteretic loops (right) for the first test of wall WDH3 and the two first tests of wall WDH5.

3.5 Modeling of the dynamic tests

Due to the aspect ratio of 4.5:1, the dynamic behaviour of the test walls was totally controlled by the fundamental mode of vibration. The analyses of the measurements could not even reveal the second natural frequency [5]. The tested walls may therefore be correctly modelled by an equivalent non-linear SDOF as shown in Figure 9. Note that for cantilever structural walls, this assumption is generally valid since the plastic deformations are concentrated at the base of the wall. It should be noted that the seismic input must be multiplied by a factor corresponding to the modal participation factor, in this case 1.30. The equation of motion was integrated using the Central Difference Method [12]. The calculations presented in this paper were performed assuming a viscous damping ratio of $\zeta=5\%$.

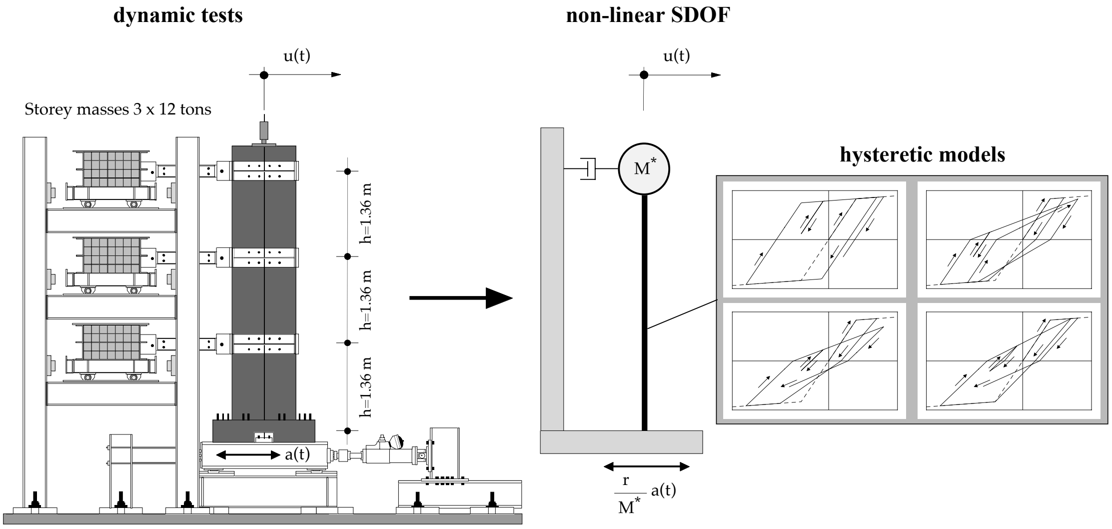


Fig. 9. Modeling of the dynamic tests using an equivalent SDOF with 4 different hysteretic models.

4 THE γ-MODEL

According to the observed feature of the hysteretic loops shown in Figure 1, the EP-model is modified with a condition for the reloading curves specified by a supplementary parameter γ. Figure 10 shows the force-displacement relationships defining the γ-model. For large yield excursions (displacements greater than current peak displacement), the reloading curves cross the elastic portion of the envelope at a height of 1-γ of the yield force F_y. Otherwise (displacements smaller than current peak displacement), the reloading curves aim for the current peak displacement. The force-displacement relationships of the γ-model are specified through four parameters: the stiffness (K), the yield displacement (u_y), the post yield stiffness (r K) and γ. The value of γ has to be determined empirically. Although theoretically the value of γ may be selected between 0 and 1, a maximum value of about γ=2/3 seems reasonable. Higher values produce excessively flat reloading curves. Similar to the EP-model, the γ-model does not consider stiffness degradation due to increasing damage. The name of the model reflects the shape of the produced hysteretic loops, which looks like the symbol γ (see Figure 10 right).

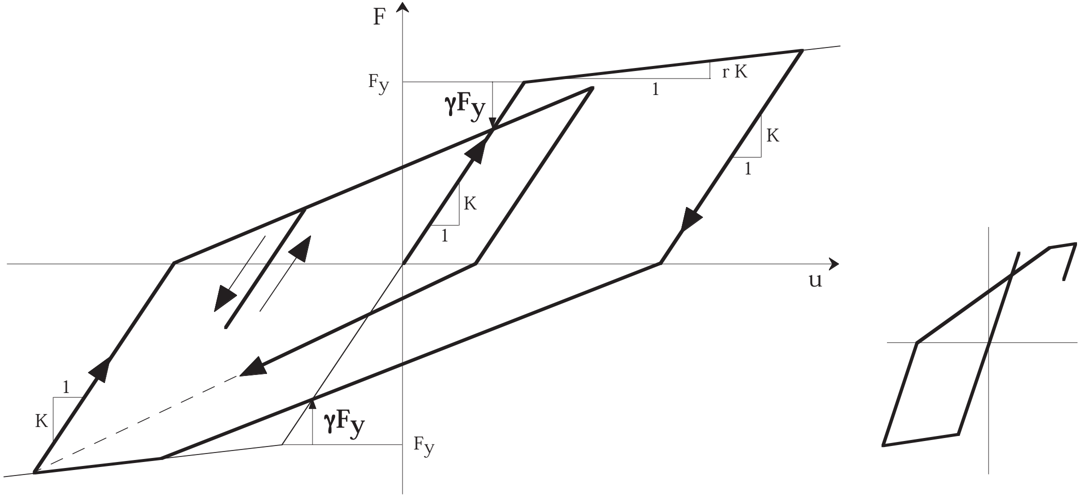


Fig. 10. Force-displacement relationships defining the γ-model.

The simplicity of the proposed rules means that the γ-model requires a significantly smaller computational effort than Takeda-model. It is estimated that the reduction in computational effort could reach a factor 10. As a consequence, the γ-model is less prone to numerical instabilities than the Takeda-model. The reliability of this simpler model is evaluated in the next section.

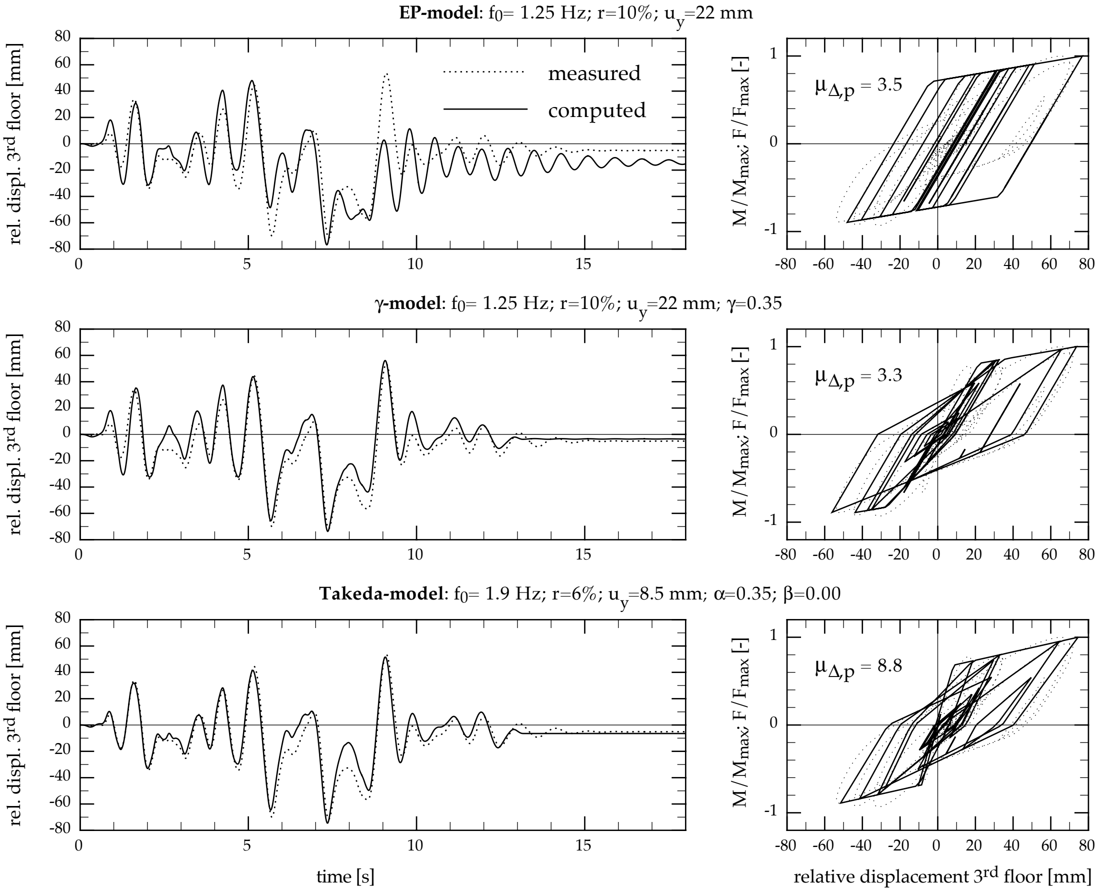
It is noted that the γ-model is based on empirical observation on RC structural walls with ductile behaviour dominated by flexure. Its applicability to non-ductile RC elements or other materials such as reinforced masonry or wood panels has not been investigated yet.

5 COMPARISON WITH ESTABLISHED HYSTERETIC MODELS

In this section the γ-model is used for a numerical simulation of the dynamic tests. The computed response is evaluated and compared with the response obtaining using the EP, Q and Takeda hysteretic models. The values of the hysteretic model parameters were selected for best match with the measured relative displacements at the 3rd floor.

5.1 WDH3 with 100% earthquake

Figure 11 shows the comparison between the EP-model, the γ-model and the Takeda-model for the first test of WDH3 with 100% earthquake. The time-histories of the relative displacements (left) and the related normalized hysteretic loops (right) are plotted. The plain lines correspond to the computed dynamic non-linear response and the dotted lines correspond to the measurements. The predicted ductility demand ($\mu_{\Delta,p}$) is also indicated.


EP-model: f_0= 1.25 Hz; r=10%; u_y=22 mm

γ-model: f_0= 1.25 Hz; r=10%; u_y=22 mm; γ=0.35

Takeda-model: f_0= 1.9 Hz; r=6%; u_y=8.5 mm; α=0.35; β=0.00

Fig. 11. Comparison between different hysteretic models for the numerical simulation of the dynamic test of WDH3 with 100% earthquake.

Figure 11 leads to the following findings:

- As expected, the measured relative displacements are poorly reproduced with the EP-model, which leads to a dynamic response strongly affected by permanent displacements.

- The Takeda-model is best at reproducing the relative displacement. Thanks to its stiffness degradation modelling ability, the Takeda-model can “track” displacements accurately in the initial “elastic” cycles as well as in the highly non-linear displacement phase of the response.
- The γ-model ability to match the displacement is only slightly inferior to the Takeda-model (except for the first four seconds)
- Best match in the highly non-linear displacement phase with the γ-model is reached with a natural frequency of f=1.25 Hz. This value may therefore be interpreted as the one of the effective fundamental frequency valid for the highly non-linear displacement phase.
- The ductility demand predicted with the γ-model $\mu_{\Delta,p}$=3.3 compares favourably with the measured value of $\mu_{\Delta,m}$=3.4 (see Figure 8).
- The best match, for the Takeda-model, is obtained with an initial natural frequency of f_0=1.9 Hz. Due to the consideration of the stiffness degradation, the effective fundamental frequency is adapted for the strong motion phase. As a consequence, the predicted ductility demand of $\mu_{\Delta,p}$=8.8 is very unrealistic [13].

5.2 WDH5 with 80% earthquake

Figure 12 shows the comparison between the γ-model, the Q-model and the Takeda-model for the first test of WDH5 with 80% earthquake.

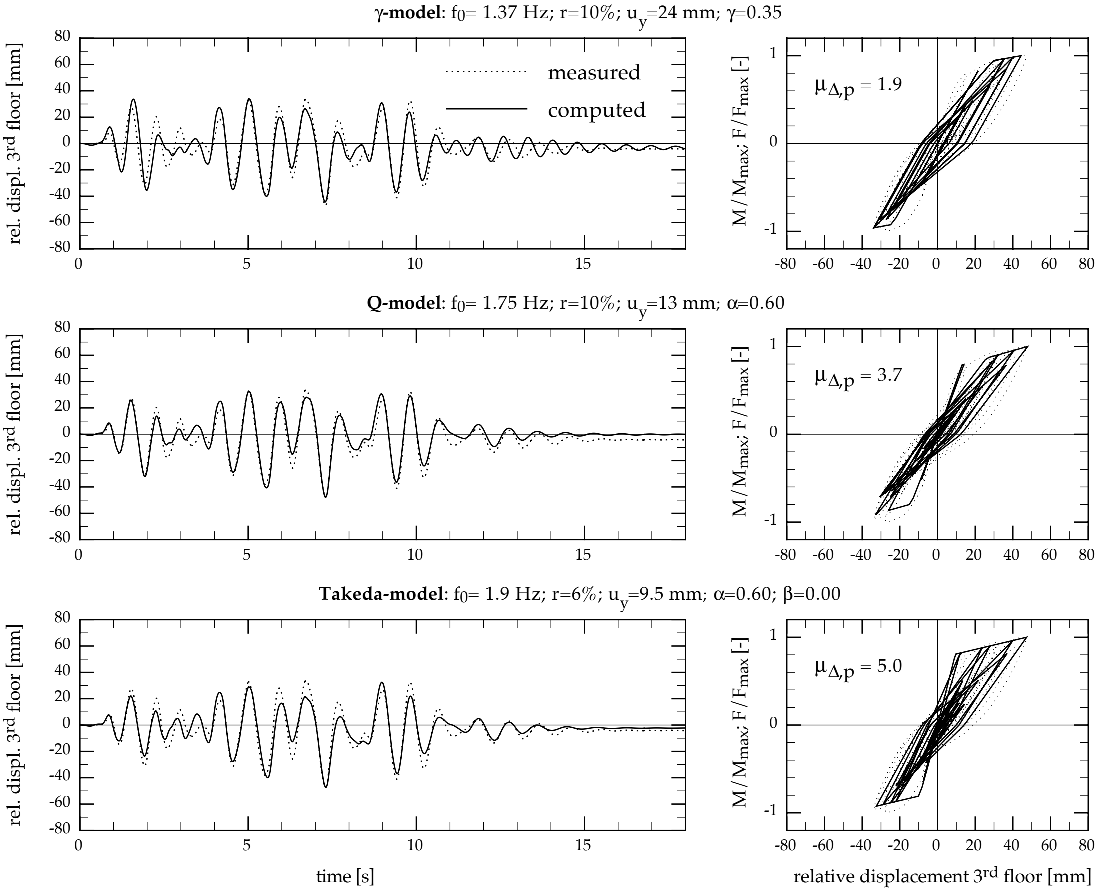


Fig. 12. Comparison between the different hysteretic models for the numerical simulation of the dynamic test of WDH5 with 80% earthquake.

Figure 12 leads to similar findings to the ones of Figure 11:

- The measured relative displacements are well reproduced with all those three hysteretic models.
- The Takeda-model and the Q-model are best at reproducing the relative displacement. Thanks to their stiffness degradation modelling ability, both models can “track” displacements accurately in the initial “elastic” cycles as well as in the highly non-linear displacement phase

of the response. In this specific case, the Q-model seems to lead to a better match than the Takeda-model.

- The γ-model ability to match the displacement is only slightly inferior to the stiffness degrading models (except for the first and the last four seconds).
- Best match in the strong motion phase with the γ-model is reached with a natural frequency of f=1.37 Hz. The ductility demand predicted with the γ-model $\mu_{\Delta,p}$=1.9 compares favourably with the measured value of $\mu_{\Delta,m}$=2.0 (see Figure 8).
- The best match, for the Q-model, is obtained with an initial natural frequency of f_0=1.75 Hz. As a consequence, the predicted ductility demand of $\mu_{\Delta,p}$=3.7 is overestimated.
- The best match, for the Takeda-model, is again obtained with an initial natural frequency of f_0=1.9 Hz. As a consequence, the predicted ductility demand of $\mu_{\Delta,p}$=5.0 is largely overestimated.

5.3 WDH5 with 100% earthquake

For this test, the initial natural frequency dropped because WDH5 was pre-damaged during the first test with 80% earthquake. Figure 13 shows the comparison between the γ-model, the Q-model and the Takeda-model in this case. In order to find similarities between both more complex models and the γ-model, the calculations are performed with the same envelope (f_0, r and u_y) and without considering stiffness degradation (α=0.0).

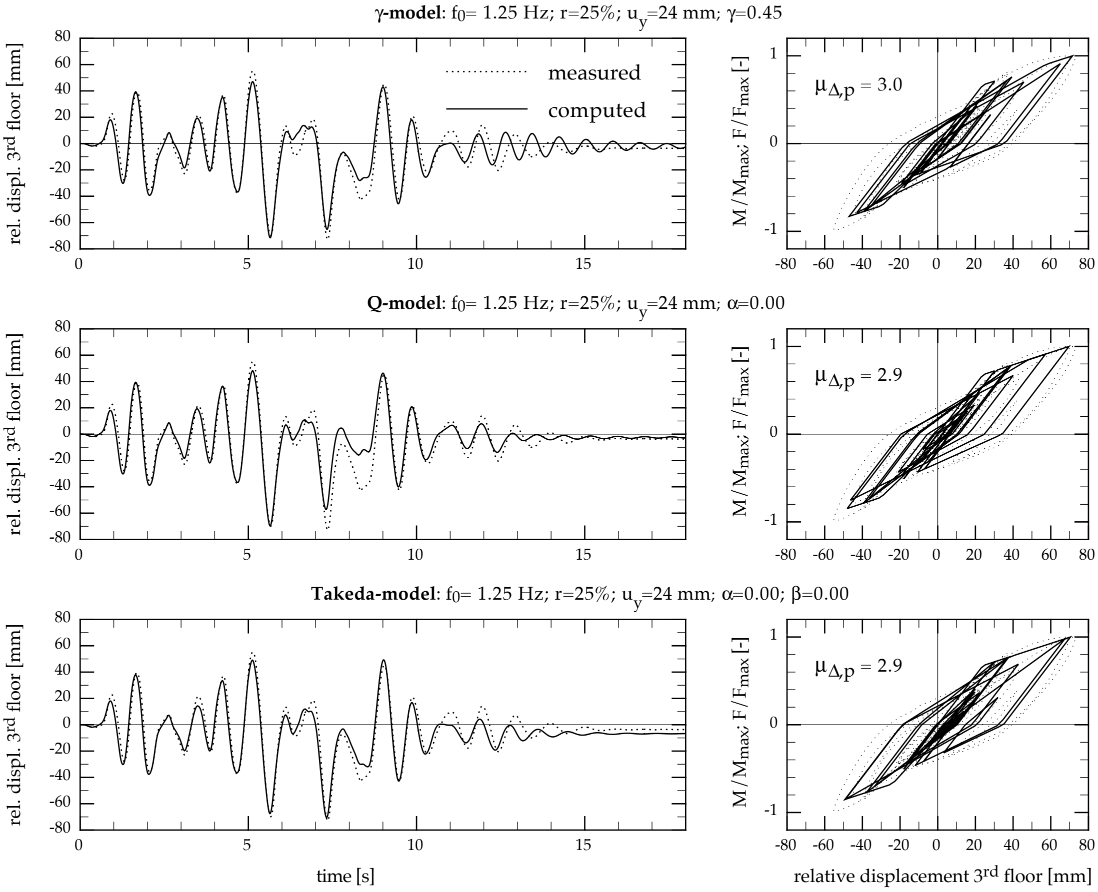


Fig. 13. Comparison between the different hysteretic models with the same envelope (f_0, r and u_y) and without considering the stiffness degradation for the numerical simulation of the second dynamic test of WDH5 with 100% earthquake.

Figure 13 leads to the following findings:

- Thanks to the reduced natural frequency of the pre-damage test wall, the measured relative displacements are very well reproduced; all three hysteretic models can "track" displacements

accurately in the initial small cycles as well as in the highly non-linear displacement phase of the response.
- The γ-model matches the displacement as well as the more complex models (except for the last few seconds).
- Best match with the γ-model is reached with a value of γ=0.45.
- In all three cases, the predicted ductility demands of $\mu_{\Delta,p}$= 2.9 to 3.0 compares favourably with the measured value of $\mu_{\Delta,m}$=3.2 (see Figure 8).

The calculated response in Figure 13 show that the γ-model may be considered as a simplified Takeda-model in which the stiffness degradation is not considered (α=0.0) and the parameter β is set to β=0.0.

6 SUMMARY AND CONCLUSIONS

A new hysteretic model, named the γ-model, is proposed. The hysteretic model was developed for RC structural walls with the objective of being able to accurately simulate their seismic behaviour with a limited computational effort. The model is based on the measured hysteretic loops of six RC structural walls recorded during dynamic tests, more specifically on the empirical observation that the reloading curves tend to cross at the same point. The force-displacement relationships of the elastoplastic model are modified in such a way that the reloading curves of large yield excursions cross the elastic portion of the envelope at a height of 1-γ of the yield force. Otherwise the reloading curves aim for the current peak displacement.

The γ-model is compared with the elastoplastic model, the Q-model and the Takeda-model. The comparison of the different hysteretic models is focused on the accurate reproduction of the measured relative displacements at the top of the wall and on the correct prediction of the ductility demand. The results of the comparison lead to the following conclusions:

- Even if it does not take into account the stiffness degradation, the γ-model ability to match the displacement in the highly non-linear phase of the response is similar to the one of the more complex Q-model and Takeda-model.
- Best matches of the displacements were obtained with values of γ between 0.35 and 0.45. This applies to RC elements with adequate seismic detailing. In order to obtain realistic hysteretic curves, the value of γ should not exceed γ=2/3.
- Because they account for the stiffness degradation, the Takeda-model and the Q-model accurately reproduce the displacements in the initial small hysteretic cycles. However, this capability is not relevant for the determination of the displacements in the highly non-linear phase. It has the negative consequence of producing overestimated predictions of the ductility demand.
- By contrast, the ductility demand predicted by the γ-model is realistic.

Due to the simplicity of its rules and the accuracy of its seismic response prediction, the γ-model provides a valuable tool for the non-linear time-history analysis of reinforced concrete structures. Compared to more complex models such as Takeda-model and Q-model, it requires significantly less computational effort. This can be important in design situations where large number of non-linear time history analyses must be conducted (i. e. design for many randomly generated design spectra compatible earthquakes). Unlike the Takeda-model, no numerical instability has been observed, i. e. the γ-model also seems to be more robust numerically.

Moreover, there exists a potentiality of an extended utilisation of the γ-model for other materials for which strength degradation does not occur under large inelastic cyclic deformations.

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