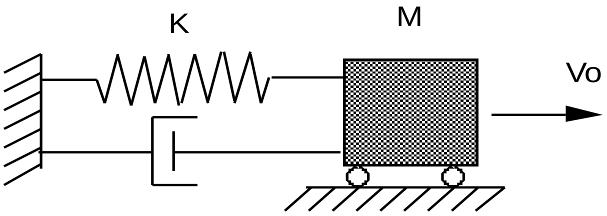
CORRIGE EXERCICE No 2

A.



Avec :
M = 20 kg
K = 20 kN/m
ζ = 0,05

Il s'agit d'un système amorti à un degré de liberté dont la solution générale est :

$$x(t) = e^{-\zeta\,\omega_n t}\left(X_0\cos(\omega_D t) + \frac{V_0 + \zeta\omega_n X_0}{\omega_D}\sin(\omega_D t)\right)$$

Conditions limites :

$X_0 = 0$

$$V_0 = \sqrt{2gh} \cong 4.43\ \mathrm{m/s}$$

Donc :

$$x(t) = e^{-\zeta\omega_n t}\left(\frac{V_0}{\omega_D}\sin(\omega_D t)\right)$$

$$\dot{x}(t) = V_0 e^{-\zeta\omega_n t}\left(-\frac{\zeta\omega_n}{\omega_D}\sin(\omega_D t) + \cos(\omega_D t)\right)$$

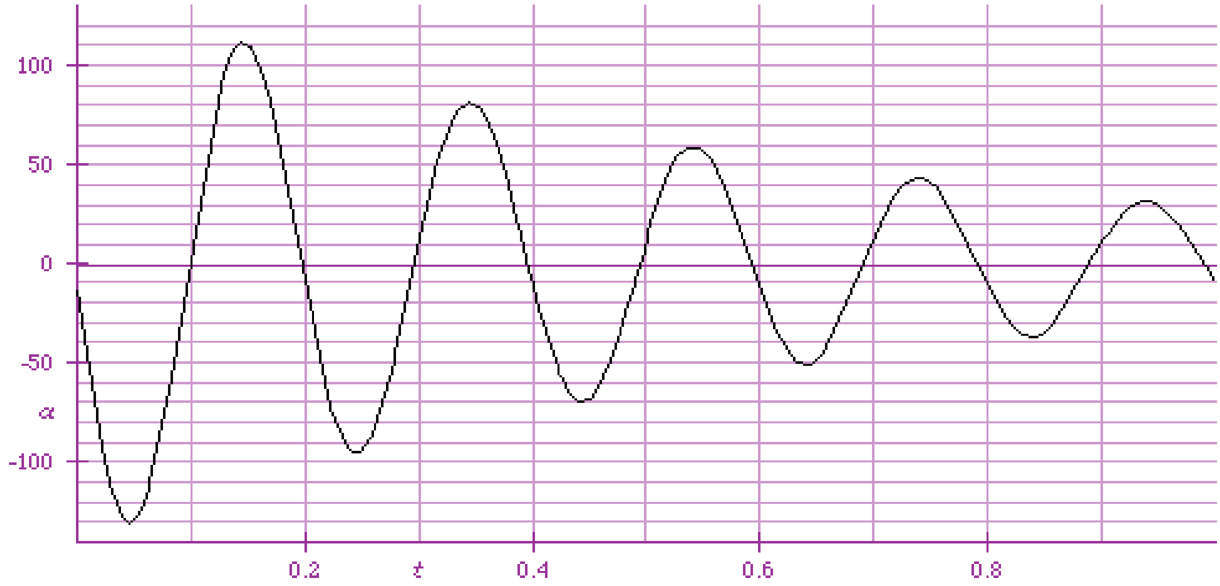
$$\ddot{x}(t) = V_0 e^{-\zeta\omega_n t}\left(\frac{(\zeta\omega_n)^2}{\omega_D}\sin(\omega_D t) - 2\zeta\omega_n\cos(\omega_D t) - \omega_D\sin(\omega_D t)\right)$$

$$\ddot{x}(t=0) = -2\zeta\omega_n V_0$$

$$\omega_n = \sqrt{\frac{K}{M}} \cong 31{,}6\ \text{rad/s}$$

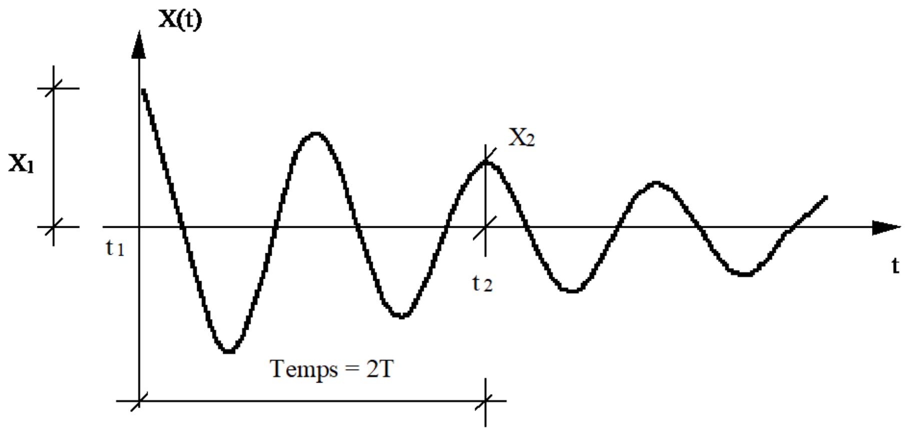
$$\zeta\omega_n = 0{,}05\ \omega_n = 1{,}58\ \ 1/s$$

$$\ddot{x}(t=0) = -2\cdot 1.58\cdot 4.43 \cong -14\frac{m}{s^2}$$



B.
C'est un système amorti à 1 degré de liberté dont la solution générale est :

$$x(t) = Ae^{-\zeta\omega_n t}\cos(\omega_D t - \phi)$$



X_1= 2,54 cm
X_2= 1,63 cm
Temps = 2·T=1.25s
T=0.625

a) Pulsation propre ω_n :

Taux de diminution de l'amplitude :

$$\delta_n = \ln(\frac{x_1}{x_2}) = \ln(\frac{2.54}{1.63}) = 0.44 \qquad \zeta = \frac{\delta_n}{2\pi \cdot n} = \frac{0.44}{2\pi \cdot 2} = 0.035$$

$$\omega_D = \frac{2\pi}{T} = 10.05\ rad/s$$

$$\omega_n = \frac{\omega_D}{\sqrt{1-\zeta^2}} = 10.06\ rad/s$$

Approximation : $\omega_n = \frac{2\pi}{T} = \frac{2 \cdot 2 \cdot \pi}{1.25} = 10.05\ rad/s$

b) Rigidité et masse :

$$K = \frac{F}{X_1} = \frac{0.890}{0.0254} = 35\frac{kN}{m} \qquad M = \frac{K}{{\omega_n}^2} = \frac{35}{10.06^2} = 0.346t \cong 346kg$$

c) Amortissement

C = $\zeta\omega_n \cdot 2 \cdot$M=244 Ns/m

C.

$$\delta_n = \ln(\frac{x_1}{x_2}) \qquad \zeta\omega_n = \frac{\delta_n}{T \cdot n} \qquad \omega_D = \frac{2\pi}{T}$$

$$\omega_n = \frac{\omega_D}{\sqrt{(1-\zeta)^2}} = 10.06\, rad/s$$

$$\zeta_n = \frac{\delta_n}{2\pi n}$$

	n	X_1	X_2	t_1	t_2	T	∂_n	$\xi\omega_n$	ω_D	ω_n	ζ	ζ_n
		[cm]	[cm]	[s]	[s]	[s]			[rad/s]	[rad/s]		
a)	1	0.80	0.60	0.6	1.9	1.3	0.288	0.221	4.831	4.836	0.046	0.046
	2	0.80	0.40	0.6	3.2	1.3	0.693	0.272	4.925	4.933	0.055	0.055
	3	0.80	0.30	0.6	4.4	1.3	0.981	0.258	4.958	4.965	0.052	0.052
b)	1	0.75	0.40	0.4	1.3	0.9	0.629	0.740	7.388	7.425	0.100	0.100
	2	0.75	0.20	0.4	2.1	0.9	1.322	0.778	7.388	7.429	0.105	0.105
	3	0.75	0.10	0.4	3	0.9	2.015	0.790	7.388	7.430	0.106	0.107

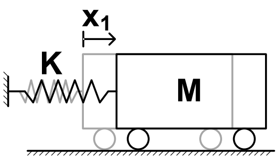
Rem.: Pour un calcul pratique, dans le cas d'un amortissement faible, on peut tirer les conclusions suivantes :

1) $\omega_D \cong \omega_n$

2) $\zeta \cong \zeta_n$

D.

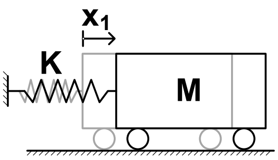
1. (a) 1 degré de liberté.
(b)



$$K = \frac{3EI}{h^3}$$

(c) $M\ddot{x}_1 + Kx_1 = 0$

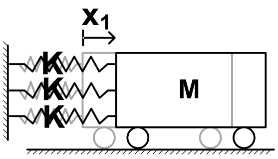
2. (a) 1 degré de liberté.
(b)



$$K = \frac{3EI}{2l^3}$$

(c) $M\ddot{x}_1 + Kx_1 = 0$

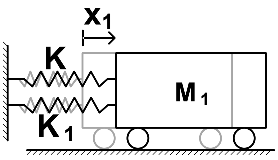
3. (a) 1 degré de liberté.
(b)



$$K = \frac{12EI}{h^3}$$

(c) $M\ddot{x}_1 + 3Kx_1 = 0$

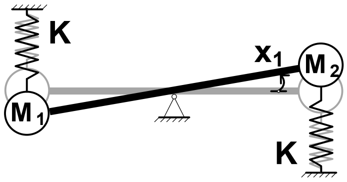
4. (a) 1 degré de liberté.
(b)

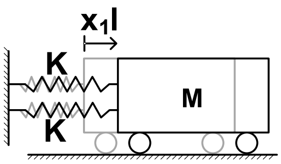


$$K_1 = \frac{3EI}{l^3}$$

(c) $M\ddot{x}_1 + (K + K_1)x_1 = 0$

5. (a) 1 degré de liberté.
(b)





$M = M_1 + M_2$

(c)

$$M\ddot{x}_1 l + 2Kx_1 l = 0 \quad \Rightarrow M\ddot{x}_1 + 2Kx_1 = 0$$